12^{th} Mathematics

Solution & Marking Scheme

1.	-	c	1
2.	-	b	1
3.	-	b	1
4.	-	d	1
5.	-	a	1
6.	-	c	1
7.	-	d	1
8.	-	d	1
9.	-	b	1
10.	-	d	1
11.	-	d	1
12.	-	c	1
13.	-	b	1
14.	-	c	1
15.	-	a	1
16.	-	d	1

17. R={(a,b): a≤b^2 }

It can be observed that $(\frac{1}{2},\frac{1}{2}) \notin$ R, since $\frac{1}{2} > (\frac{1}{2})^2$ 1

∴ R is not reflexive.

Now (1, 4) ∈ R as $1<4^2$ but 4 is not less than 1^2

∴ (4,1) ∉ R

∴ R is not symmetric. 1

Now (3, 2) (2, 1.5) ∈ R but 3> $(1.5)^2$ = 2.25

∴ (3, 1.5) ∉ R So R is not transitive. 1

Hence R is neither reflexive nor symmetric

Nor transitive. (1+1+1=3)

18. Here aij = $\frac{(i+j)^2}{2}$

So, $a_{11}=\frac{(1+1)^2}{2}$=2 $\quad a_{12}=\frac{(1+2)^2}{2}=\frac{9}{2}$ 2

a_{21} $\frac{(2+1)^2}{2}=\frac{9}{2}$ $\quad a_{22}=\frac{(2+2)^2}{2}=8$

So matrix= $\begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$ 1

(2+1=3)

19. The area of Triangle is given by

$\triangle = \frac{1}{2}\begin{bmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{bmatrix}$ 1

$= \frac{1}{2}$ [3 (2-1) -8 (-4-5) + 1(-4-10] 1

$=\frac{1}{2}$ (3 + 72-14)= $\frac{61}{2}$ (1+1+1=3)

20. Given that the function is continuous at x=2

So L.H.L + R.H.L. = f (2)

$\Rightarrow x \xrightarrow{\lim} 2^-$ f(x) = $x^{lt} \to 2^t$ f (x) = f (2) 1

$\Rightarrow x \xrightarrow{\lim} 2^-$ (kx^2) = $x^{lt} \to 2^t$ 3 = $k(2)^2$ 1

$\Rightarrow 4k = 3$ 1

$\Rightarrow k = \frac{3}{4}$ $(1 + 1 + 1 = 3)$

21. $\int \frac{1-\cos x}{1+\cos x}$ dx

$=\int \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} dx = \int \tan^2\frac{x}{2}$ dx 1

$= \int (\sec^2\frac{x}{2} - 1)$ dx 1

$= \frac{\tan\frac{x}{2}}{\frac{1}{2}} - x + c$ 1

$= 2\tan\frac{x}{2} - x + c$ $(1 + 1 + 1 = 3)$

22. $I = \int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}$ dx ---------- 1

$I = \int_0^{\frac{\pi}{2}} \frac{\sin^4(\frac{\pi}{2}-x)}{\sin^4(\frac{\pi}{2}-x) + \cos^4(\frac{\pi}{2}-x)}$ 1

$= \int_0^{\frac{\pi}{2}} \frac{\cos^4 x}{\cos^4 x + \sin^4 x}$ ---------- 2 1

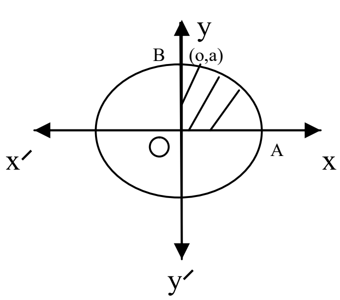
Adding eq. 1 and eq.2

$2I = \int_0^{\frac{\pi}{2}} \frac{(\sin^4 x + \cos^4 x)}{(\sin^4 x + \cos^4 x)}$ dx $\Rightarrow \int_0^{\frac{\pi}{2}} 1dx \Rightarrow [x]_0^{\frac{\pi}{2}}$

$2I = \frac{\pi}{2}$ 1

$\Rightarrow I = \frac{\pi}{4}$ $(1 + 1 + 1 = 3)$

23.



1

Required area = 4 (area of region AOBA)

$= 4\int_0^a y dx$

$= 4\int_0^a \sqrt{a^2 - x^2}\ dx$ 1

$= 4\left[\frac{x}{2}\sqrt{a^2-x^2}\ +\frac{a^2}{2}\ sin^{-1}\ \frac{x}{a}\right]$

$= 4\ \left[\left(\frac{a}{2}X0+\frac{a^2}{2}\ sin^{-1}(1)\right)-0\ \right]$

$= 4\ (\frac{a^2}{2})(\frac{\pi}{2}) =\ \pi a^2$ 1 (1+1+1=3)

24. $\frac{dy}{dx}+2y = \text{sinx}$

Here comparing with $\frac{dy}{dx} + \text{Py} = \ Q$

Where P = 2 & Q = sin x

So I.F. = $e^{\int Pdx} = e^{\int 2d}$ =e^{2x} 1

So solution of differential equation is

y (I.F.) = $\int (\ Q \text{ x } I.F.)dx + c$

$\Rightarrow$ y e^{2x}= $\int$ sin x. $e^{2x}\ dx + c$ ---------- 1 $\frac{1}{2}$

Let I = $\int$ sin x. e^{2x} dx

$=\sin x \int\ e^{2x}\ dx - \int\left[\frac{d}{dx}(sinx).\int e^{2x}\ dx\right]dx$

I = sin x.$\frac{e^{2x}}{2}\ -\ \int cosx.\frac{e^{2x}}{2}\ dx$

$=\frac{e^{2x}.sinx}{2}-\frac{1}{2}\int cosx.e^{2x}dx$ $\frac{1}{2}$

$=\frac{e^{2x}.sinx}{2}-\frac{1}{2}[cosx.\frac{e^{2x}}{2}-\int\left[-sinx\frac{e^{2x}}{2}\right]dx]$

$=\frac{e^{2x}.sinx}{2}-\ \frac{e^{2x}.cosx}{4}$ - $\frac{1}{4}\int\ sinx\frac{e^{2x}}{2}\ dx$

I= $\frac{e^{2x}}{4}$(2sinx-cosx)- $\frac{1}{4}I$

$\Rightarrow \frac{5}{4}$ I=$\frac{e^{2x}}{4}$(2sinx-cosx)

$\Rightarrow$ I = $\frac{e^{2x}}{5}$(2sinx-cosx)

y e^{2x} = $\frac{e^{2x}}{5}$(2sinx-cosx)+c

$\Rightarrow$ y = $\frac{1}{5}$ (2sinx-cosx)+c e^{-2x} 1

$(1+\frac{1}{2}+\frac{1}{2}+1=3)$

Or

$\frac{dy}{dx}=\frac{1-cosx}{1+cosx}$

$=\ \frac{2sin^2\frac{x}{2}}{2cos^2\frac{x}{2}} \Rightarrow \tan^2\frac{x}{2}$ 1

$\Rightarrow \frac{dy}{dx} = (\sec^2\frac{x}{2}-1)$

Separating variables

dy = $(\sec^2\frac{x}{2}-1)$ dx 1

Now, integrating both sides

$\int$ dy = $\int (\sec^2\ \frac{x}{2}-1)$dx

$= \int \sec^2\frac{x}{2}dx - \int 1dx$

$y = 2\tan\frac{x}{2} - x + c$ 1

Which is the required general solution. (1+1+1=3)

25. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$ 1

$= \hat{\imath}(-2-15) - (-4-9)\hat{\jmath} + (10-3)\hat{k}$ 1

$= -17\hat{\imath} + 13\hat{\jmath} + 7\hat{k}$

So $|\vec{a} \times \vec{b}| = \sqrt{(-17)^2 + (13)^2 + (7)^2}$ 1

$= \sqrt{507}$ (1+1+1=3)

26. let $\underset{OA}{\rightarrow} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\underset{OB}{\rightarrow} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$

$\underset{OC}{\rightarrow} = 3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}$ 1

Now vector $\underset{AB}{\rightarrow}, \underset{BC}{\rightarrow}$ *and* $\underset{AC}{\rightarrow}$ represent the sides of $\triangle$ABC

i.e. $\underset{AB}{\rightarrow} = (1-2)\hat{\imath} + (-3+1)\hat{\jmath} + (-5-1)\hat{k} = -\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$

$\underset{BC}{\rightarrow} = (3-1)\hat{\imath} + (-4+3)\hat{\jmath} + (-4+5)\hat{k} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\underset{AC}{\rightarrow} = (2-3)\hat{\imath} + (-1+4)\hat{\jmath} + (1+4)\hat{k} = -\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$ 1

$|\underset{AB}{\rightarrow}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{1+4+36} = \sqrt{41}$

$|\underset{BC}{\rightarrow}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4+1+1} = \sqrt{6}$

$|\underset{AC}{\rightarrow}| = \sqrt{(-1)^2 + 3^2 + 5^2} = \sqrt{1+9+25} = \sqrt{35}$

Here $|\underset{BC}{\rightarrow}|^2 + |\underset{AC}{\rightarrow}|^2 = 6+35 = 41 = |\underset{AB}{\rightarrow}|^2$ 1

$\triangle$ABC is a right angled triangle. (1+1+1=3)

27. n (R) =8 , n (B) = 10

Total Balls = 10+8=18

P (Red ball) = $\frac{8}{18} = \frac{4}{9}$

1

The ball is replaced after Ist draw

P (again a red ball) = $\frac{8}{18} = \frac{4}{9}$ 1

So P (both balls are red) = $\frac{4}{9} X \frac{4}{9} = \frac{16}{81}$ 1 (1+1+1=3)

28. $P(E_1) = P(E_2) = \frac{1}{2}$

Let A be the event of getting a Red ball. 1

$\Rightarrow P(A|E_1) = \frac{4}{8} = \frac{1}{2}$

$\Rightarrow P(A|E_2) = \frac{2}{8} = \frac{1}{4}$ 1

Using Bayes' Theorem:

$$P(E_1|A) = \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)}$$

$$= \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{4}}$$

$= \frac{2}{3}$ 1 (1+1+1=3)

Or

S = {HHH, HHT, THH, HTH, TTH, THT, HTT, TTT}

N (S) = 8

Here E = {HHH, THH, HTH, TTH}

F = {HHT, HHH}

$E \cap F =$ {HHH}

1

$P(F) = \frac{2}{8} = \frac{1}{4}$ 1

and $P(E \cap F) = \frac{1}{8}$

Now $P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{1}{8} \Big/ \frac{1}{4} = \frac{1}{2}$ 1 (1+1+1=3)

Section-C

29. L.H.S. $= \cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right)$

$= \sin^{-1}\sqrt{1 - \left[\frac{12}{13}\right]^2} + \sin^{-1}\left(\frac{3}{5}\right)$ 1

$= \sin^{-1}\sqrt{\frac{169-1}{169}} + \sin^{-1}\left(\frac{3}{5}\right)$

$= \sin^{-1}\frac{5}{13} + \sin^{-1}\left(\frac{3}{5}\right)$ 1

$= \sin^{-1}\left\{\frac{5}{13}\sqrt{1 - \left[\frac{3}{5}\right]^2} + \frac{3}{5}\sqrt{1 - \left[\frac{5}{13}\right]^2}\right\}$ 1

$= \sin^{-1}\left\{\frac{5}{13}\sqrt{1 - \frac{9}{25}} + \frac{3}{5}\sqrt{1 - \frac{25}{169}}\right\}$

$= \sin^{-1}\left\{\frac{5}{13} X \frac{4}{5} + \frac{3}{5} X \frac{12}{13}\right\}$

$= \sin^{-1}\left(\frac{20+36}{65}\right) = \sin^{-1}\frac{56}{65}$ = R.H.S. 1 (1+1+1+1=4)

Or

$tan^{-1}\left(\frac{cosx-sinx}{cosx+sinx}\right)$ Dividing each term by cos x 1

$\Rightarrow\ tan^{-1}\left[\frac{1-\frac{sinx}{cosx}}{1+\frac{sinx}{cosx}}\right] \Rightarrow\ tan^{-1}\left[\frac{1-tan\,x}{1+tan\ x}\right]$ 1

$\Rightarrow\ tan^{-1}\left[tan\left(\frac{\pi}{4}-x\right)\right]$ 1

$\Rightarrow \frac{\pi}{4}-x$ 1 (1+1+1+1=4)

30. xy+y^2 = tanx +y

Diff. both sides W.r.t. x

$\Rightarrow x\frac{dy}{dx}+y.1+2y\frac{dy}{dx} = sec^2x+\frac{dy}{dx}$ 1

$\Rightarrow x\frac{dy}{dx}+2y\frac{dy}{dx} - \frac{dy}{dx} = sec^2x - y$ 1

$\Rightarrow (x+2y-1)\frac{dy}{dx} = sec^2x-y$ 1

$\Rightarrow \frac{dy}{dx} = \frac{sec^2 x-y}{x+2y-1}$ 1 (1+1+1+1=4)

Or

$(cosx)^y = (cosy)^x$

Taking log both sides.

y log cosx = x log cosy 1

Diff. both side w.r.t.x

$\Rightarrow y.\frac{1}{cosx}(-sinx)+\log cosx\frac{dy}{dx}=x.\frac{1}{cosy}(-siny)\frac{dy}{dx}+\log cos\,y$ 1

$\Rightarrow \frac{dy}{dx}[\log cosx + xtany]$ = log cosy +ytan x 1

$\Rightarrow \frac{dy}{dx} = \frac{\log cos\ y + y\,tan\ x}{log\ cos\,x + x\,tan\,y}$ 1 (1+1+1+1=4)

SECTION-D

31. A = $\begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}$ 1

$\Rightarrow |A| = 40 \neq 0$

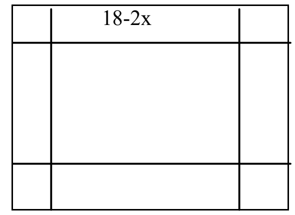
$A^{-1} = \frac{1}{|A|}$ adj A = $\frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix} \Rightarrow$ X = A^{-1} B 1

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$ 2

X=A^{-1} B

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \Rightarrow$ x=1, y=2, z = -1 1 (1+1+2+1=5)

32. Let side of square to be cut off be x cm then the length and the breadth of the box will be (18-2x) cm each and the height of the box is x cm.



1

So v (x) = x $(18-2x)^2$ 1

v′ (x) = $(18-2x)^2$-4 $(18-2x) \Rightarrow (18-2x)\ [18-2x-4x]$

= $(18-2x)\ (18-6x)$ ⇒6x2 $(9-x)\ (3-x)$

= v′′ (x) = 12 $[-(9-x)-3(3-x)]$ 1

= -12 $(9-x+3-x) \Rightarrow$ -12 (12-2x) ⇒ -24 (6-x) < 0

Now v′ (x) =0 ⇒ x=9 (Not possible) or x=3 1

So x=3 is the point of maxima of V

Hence, if we cut a square of side 3 cm from each side of corner volume of box obtained is largest possible. 1 (1+1+1+1+1=5)

Or

For each value of x, the helicopter position is at point (x, x^2+7) so the distance between the helicopter and the soldier placed at (3,7) is

$\sqrt{(x-3)^2+(x^2+7-7)^2} \Rightarrow \sqrt{(x-3)^2+x^4}$ 1

Let f (x) = $(x-3)^2$+x^4

Put = f′ (x) =0 1

⇒ x=0 or $2x^2$+2x+3=0 for which there are no real roots. Also there are no end points of the interval so that f is zero. i.e. there is only one point, x=1 1

So f (1) = $(1-3)^2$+$(1)^4$=5

So distance between soldier and helicopter is $\sqrt{f(x)} = \sqrt{5}$ 1

$\sqrt{f(0)} = \sqrt{(0-3)^2+(0)^4}$ = 3 > $\sqrt{5}$

So $\sqrt{5}$ is the minimum value of $\sqrt{f(x)}$. Hence $\sqrt{5}$ is the minimum distance between the soldier and the helicopter. 1 (1+1+1+1+1=5)

33. $\overrightarrow{a_2}$ - $\overrightarrow{a_1}$ = $3\hat{\imath}+3\hat{\jmath}+3\hat{k}$

$\overrightarrow{b_1}$ x $\overrightarrow{b_2}$ = $\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$ 1

$\overrightarrow{b_1}$ x $\overrightarrow{b_2}$ = $-9\hat{\imath}+3\hat{\jmath}+9\hat{k}$

$\left|\overrightarrow{b_1} \text{ x } \overrightarrow{b_2}\right| = \sqrt{181+9+81} = \sqrt{171}$ 1

S.D. = $\left|\frac{(3\hat{\imath}+3\hat{\jmath}+3\hat{k}).(-9\hat{\imath}+3\hat{\jmath}+9\hat{k})}{\sqrt{171}}\right|$ 2

= $\left|\frac{-27+9+27)}{\sqrt{171}}\right| = \frac{9}{3\sqrt{19}} = \frac{3}{\sqrt{19}}$ units. 1 (1+1+2+1=5)

Or

Here $\vec{b_1} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{b_2} = 3\hat{\imath}+2\hat{\jmath}+6\hat{k}$ 1

So, cos $\theta = \left|\frac{\vec{b_1}\cdot\vec{b_2}}{\left|\vec{b_1}\right|\left|\vec{b_2}\right|}\right| \Rightarrow \left|\frac{(\hat{\imath}+2\hat{\jmath}+2\hat{k}).(3\hat{\imath}+2\hat{\jmath}+6\hat{k}}{\sqrt{1+4+4}.\sqrt{9+4+36}}\right|$ 2

$= \left|\frac{3+4+12}{3 \; X \; 7}\right| = \frac{19}{21}$ 2

Hence, $\theta = cos^{-1} \left(\frac{19}{21}\right)$ (1+2+2=5)

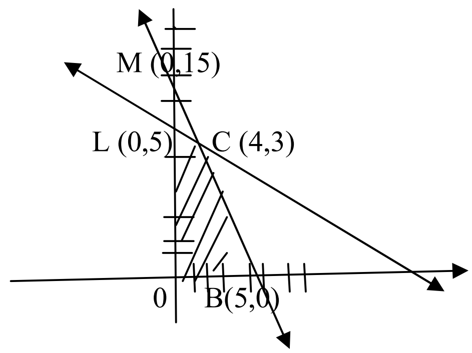
34. given x,y ≥ 0

x+2y≤ 10 2

3x+y ≤ 15

Maximum

Z=3x+2y



Solution:-

	Z=3x+2y
0 (0,0)	Z = 0+0=0
B (5,0)	Z = 3(5) + 0=15
C (4,3)	Z = 3(4) +2 (3) =18
L (0,5)	Z = 3(0) + 2 (5) = 10

2

The maximum value = 18 at (4,3) 1 (1+2+2=5)